# Stress-strain relationships in the analysis of the failure process of weak rock masses

## Spannung-Entstellung Beziehungen im Ausbruchvorganganalyse von weichem Gebirge

## Relations contraintes-déformations dans l'analyse du processus de rupture de massifs rocheux tendres

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ABSTRACT: The special characteristics of weak rocks make necessary the consideration of the stress-strain relationships to carry out the study of their behaviour and deformational process, since these relations control the strength and behaviour of this type of materials. The use of failure criteria considering the failure beginning before the peak strength is reached, instead of those for brittle material, has shown appropiate for the correct study and modelling of weak rock masses. In this paper, the study of the stress-strain relationships of carboniferous shales of an open pit slope is carried out. From the strain values measured in the laboratory and from the shape of the stress-strain curves, different behaviour patterns can be deduced for the material, which have been used as failure criteria for the stability analysis of the slope.

RESUMÉ: Les caractéristiques particulières des roches tendres rendent nécessaire la considératión de leurs relations contraintes-déformation à l'occasion de l'étude de leurs comportements et de leurs processus de déformation, étant donné que ce sont précisément ces relations celles qui contrôllent la résistance et le comportement de ce type de matériaux. L'utilisation de critères de rupture qui considèrent que celle-ci commence avant d'atteindre la resistance maximale, au lieu de ceux qui sont communnément utilisés pour les matériaux fragiles, s'est révélée adéquate pour l'étude et la modélisation correctes des massifs rocheux tendres. On réalise dans ce travail, une étude des relations contraintes-déformations des lutites carboniféres d'un talus minier. A partir des valeurs des déformations obtenues au laboratoire, et à partir de la morfologie des courbes contraintes-déformations, on obtient diférents modèles de comportement du matériau, qui sont utilisés come critères de rupture dans l'analyse de la stabilité du talus.

ZUSAMMENFASSUNG: Die besonderen Kennzeichungen von weichen Gesteinen erfordern seinen Spannung-Entstellung Beziehungen anzuweisen, um die Analyse seiner Betragen und Entstellungsvorgange auszuführen, da diese Bezienhungen die Festigkeit und das Betragen diese Stoffe kontrollieren. Die Verwendung von Ausbruchkriteriums, die betrachten, dass der Ausbruch beginn, bevor die Spitzfestigkeit sich ereignet, anstatt der gewöhnlich für weiche Stoffe gebrauchten Kriteriums, ist angemessen für die richtige Analyse und Vorbildung weiches Gebirge. In dieser Arbeit findet die Analyse von den Spannung-Entstellung Beziehungen einer kohlenhaltiger Lutiten in einer bergmännischer Böschung statt. Mit den im Laboratorium gemessen Entstellungswerten und mit der Form der Spannung-Entstellung Kurven sind verschiedene Betragensvorbilde des Stoffes erlangt, die als Ausbruchkriterium in der Analyse der Beständigkeit der Böschung verwendet sind.

## 1 INTRODUCTION

The study of rock masses in which joints play the main rol in the instability and failure processes, has been based, basically, on failure criteria applied to brittle materials with an assumed elastic behaviour.

The use of this type of criteria is not appropiated for the investigation of weak rocks, which don't present a brittle behaviour and for which the beggining of the failure can not be associated, in general, to the peak strength. Furthermore, the failure mechanisms of this type of rocks are not basically controlled by the joint sets of the rock mass.

The study presented here started from the observation of the behaviour of an open pit slope in shales, which suffered plasticity processes and ductile deformations when a certain depth were reached; the general slope resulted completely failed as a consequence of these processes.

On analysing the stability of the slope by limit equilibrium methods, after having characterized the material with conventional laboratory tests, and assuming failure mechanisms for jointed rock masses, the obtained results indicated that the slope could not be unstable. The analysis didn't respond to the real behaviour and it didn't allow to modelize adequately the observed deformational process.

These results gave rise to the detailled study of the behaviour of the weak rock mass, basically based on the stress-strain relationships of the material.

The investigation was specially focussed to the validity of the failure criteria, the importance of the strains suffered by the material and their influence on the strength, the different behaviour of joints and intact rock and the elastic and plastic stress-strain relations. The main objective was to explain and to interpret the observed behaviour and deformational process of the rock mass, as well as to know the parameters governing and influencing the strength of the material.

## 2 STUDIED CASE

The deformational and failure process of the rock mass, which went on for more than a year, showed a ductile-like behaviour of the material, mainly characterized by the slow deformation of the toe of the slope, with flexion and buckling of the beds, until it broke resulting the material completely destroyed and provoking the general instability of the slope (see Figure 1).

After the field work, with the collection of data and measures, the execution of boreholes and laboratory tests, the geological and hidrogeological patterns were established (Figure 2).

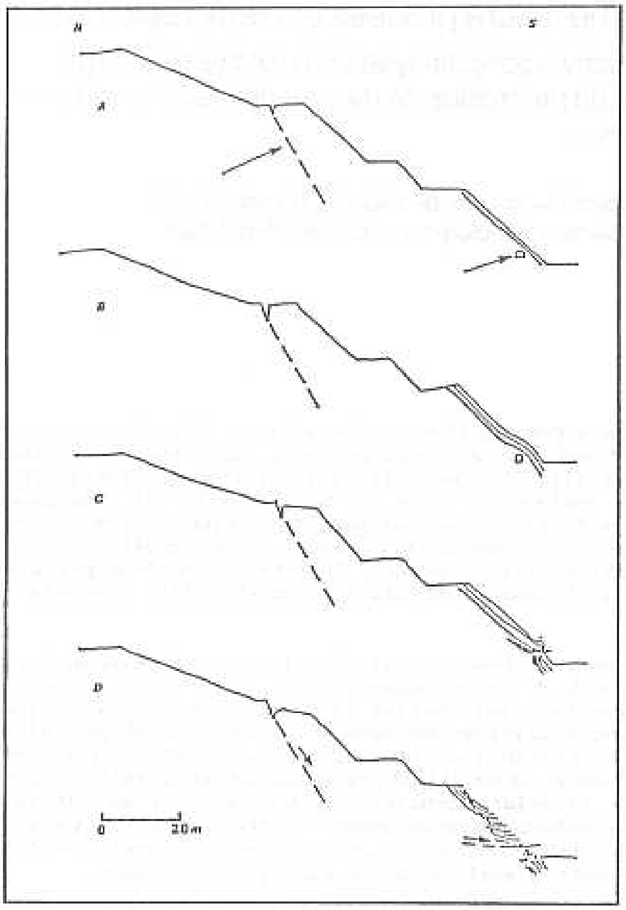


Figure 1. Steps of the evolution of the deformational process of the slope, showing the ductil behaviour of the toe.

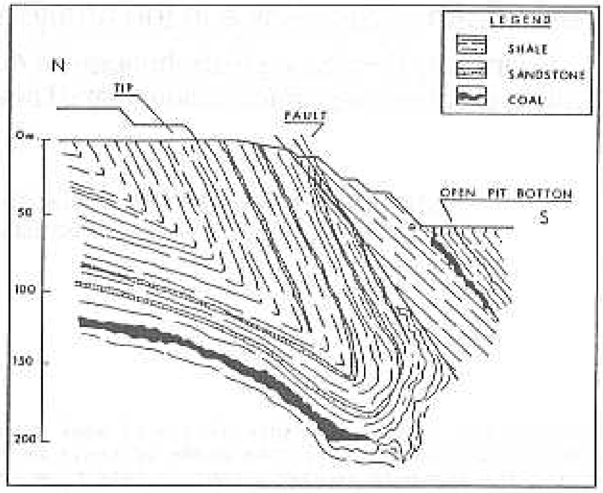


Figure 2. Geological model of the slope.

The data obtained from the boreholes, and from the field observations and measures, allowed to deduce the failure model: it was a curve failure with a big tension crack (coinciding with a fault detected in the upper part of the slope) and coming out at the toe of the slope.

In the upper part of the failure surface, the acting geomechanical parameters would be those of the own fault, but, from the point this surface went off the fault, it was no conditioned by any of the joint sets measured in the slope, but the failure took place through the intact rock, almost perpendicular to the bedding planes.

The limit equilibrium analysis, carried out with data collected in the field and from laboratory tests, showed the circular failure as the most feasible. However the back-analysis showed for cohesion and friction much more lower values than those obtained from the laboratory. When the data obtained from tests were used to characterize the fault and the intact rock in the analysis, the slope resulted always in a stable situation.

As the problem could not be solved, the study of the deformational mechanisms and the behaviour law of the material was carried out in order to explain and to modelize the failure process.

The main problem arised was the knowledge of the adequate behaviour law and failure criteria. While for hard rock masses this problem is solved on working with the strength of joints, for weak rocks the failure surfaces can be generated through intact rock, which deformational process responds to a ductile or plastic behaviour that are not considered by the conventional methods used in the analysis of hard rock masses (Elliot and Brown, 1985).

## 3 STUDY OF THE STRESS-STRAIN RELATIONSHIPS IN LABORATORY TESTS

In order to study the stress-strain relationships of the material and to get the deformational and strength parameters, some uniaxial compression tests were carried out with a stiff servo-controlled testing machine, which allowed the record of the complete stress-axial/radial strain curves, including the post-peak region, due to the stiff machine/stiff specimen relation (Wawersik and Fairhurst, 1970; Brady and Brown, 1985).

By considering the shape of the stress-axial strain curves, the specimens could be classified into three groups:

- curves which show a rather brittle behaviour with a marked peak failure ("brittle"failure),
- curves with a marked peak failure, without pre-peak failure, and with a more developed pos-peak portion ("brittle-ductile" behaviour), and
- curves which show a pre-peak failure and an important developement of the post-peak portion ("ductile" behaviour).

In Figure 3 these three types of curves have been represented, using the average values of the stress and axial strain data for each group.

Figures 4 and 5 show the strength and strain values obtained for the tested specimens, including those values corresponding to the peak, to the yield point and to residual values. The values of the angle between the bedding planes and the axial load direction have been also represented in Figure 5.

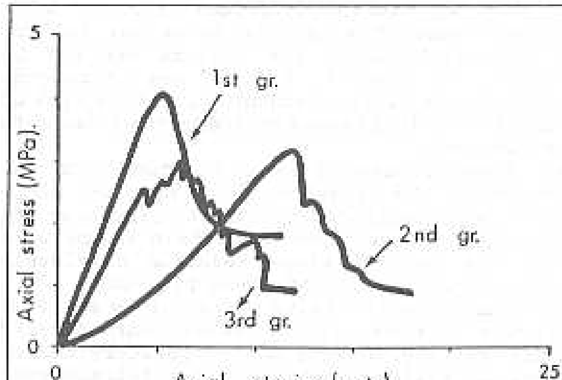


Figure 3. Representative curve patterns obtained for the tested specimens, corresponding to brittle (1st group), brittle-ductile (2nd group) and ductile-like behaviour (3rd group).

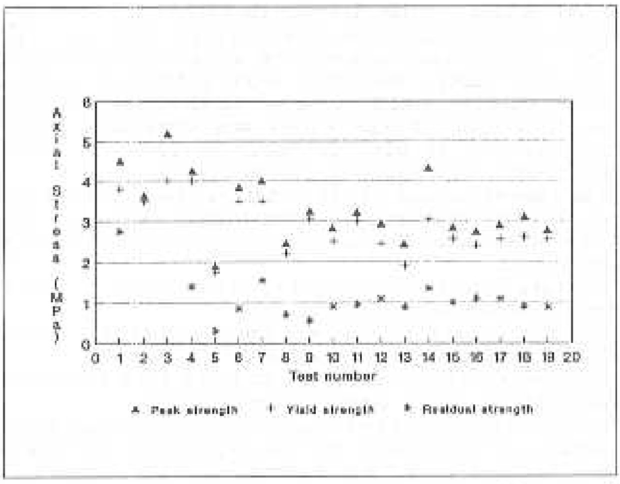


Figure 4. Axial stresses measured for the specimens. No conclussions can be stablished to indicate differences in the behaviour of the specimens.

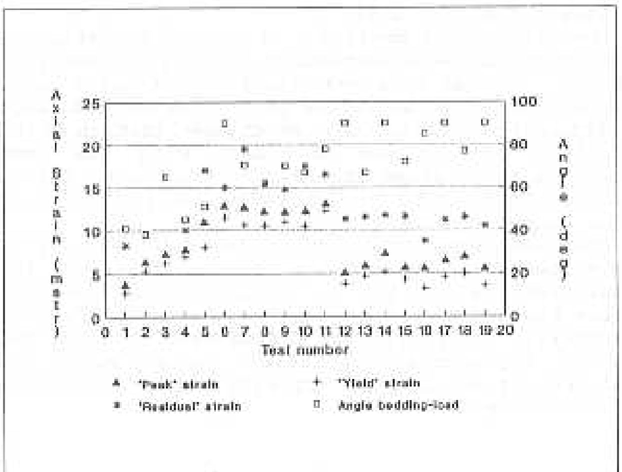


Figure 5. Axial strains measured for the specimens, in which three different groups can be seen.

With respect to the uniaxial peak strength of the specimens (Table 1 shows the mean values for the three groups), no conclusions can be obtained to stablish different behaviours of the material; the values, between 2 and 5 MPa, are much more lower than those obtained for the same type of rocks with conventional laboratory tests (25-30 MPa; Ferrer, 1990). The values corresponding to the yield strength vary from 1.8 to 4 MPa, being the mean value 0.42 MPa less than the mean peak strength, and the residual strength values are between 0.3 and 1.55 MPa (excluding the value of 2.75 MPa for test nº 1).

Table 1. Mean strength values obtained from the laboratory tests.

| | $\sigma_c$(MPa) | $\sigma_y$(MPa) | $\sigma_r$(MPa) |
|---|---|---|---|
| 1st group | 4.36 | 3.82 | 2.10 |
| 2nd group | 3.10 | 2.78 | 0.83 |
| 3rd group | 3.00 | 2.51 | 1.04 |

From the axial strain values of Figure 5, a clear differentiation is deduced: three groups can be clearly distinguished (test numbers 1-4, 5-11 and 12-19). These axial strain values (Table 2) show the different behaviour of the specimens, which was also showed by the shape of the stress-axial strain curves: the tests of each of the three groups present similar axial strain values and the same curve pattern.

Table 2. Mean axial strain values obtained from the laboratory tests.

| | $\epsilon_c$(mstr) | $\epsilon_y$(mstr) | $\epsilon_r$(mstr) |
|---|---|---|---|
| 1st group | 6.20 | 5.29 | 9.11 |
| 2nd group | 12.24 | 10.61 | 16.8 |
| 3rd group | 5.93 | 4.31 | 11.0 |

Some conclussions can be also obtained attending to the load-bedding angle: for the first group ("brittle" behaviour) this angle vary between 38º and 45º (except for a specimen with 65º); for the second group, most of values are between 60º and 70º; and for the third group ("ductile" behaviour), values vary specially from 76º to 90º.

These previous considerations reveal that the different behaviours of the tested specimens can be deduced from the axial strain values, being not valid, in this case, the strength values to get conclussions with respect to the behaviour of the material (for all the tests the obtained strength values are very similar). In the same way can be said that the strain ranges control the behaviour and strength of the material.

The failure patterns of the specimens also show the differences between the "brittle" group (failing along bedding planes) and the "ductile-like" group, where the specimens resulted almost completely destroyed failing across a great number of generated failure planes (Figure 6).

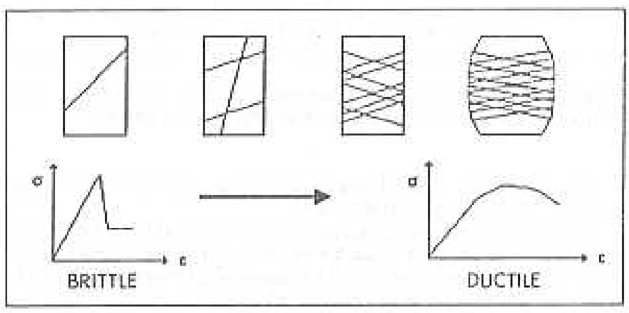


Figure 6. Different failure patterns obtained for the tested specimens.

In order to characterize the behaviour of the tested rocks, and considering that most of the specimens showed a ductil behaviour, the failure criterion deduced was to consider that failure is reached when the specimen can no longer adequately support the loads applied to it without suffering non recoverable deformation: that is, the failure begin at the yield point. From the axial strain values, this point occur for:

1st. group: $\epsilon_y$ = 5.3 mstr = 86% $\epsilon_c$ and = 58% $\epsilon_r$
2nd. group: $\epsilon_y$ = 10.6 mstr = 86% $\epsilon_c$ and = 62% $\epsilon_r$
3rd. group: $\epsilon_y$ = 4.3 mstr = 73% $\epsilon_c$ and = 39% $\epsilon_r$

being

$\epsilon_y$ = axial strain at the yield point
$\epsilon_c$ = axial strain at the peak strength
$\epsilon_r$ = axial strain when the residual strength is reached.

The more ductile is the behaviour of the material, the more difference can be measured between $\epsilon_c$ and $\epsilon_y$ and between $\epsilon_c$ and $\epsilon_r$.

With respect to the Young's modulus, E, and the Poisson relation, $\nu$, the mean values are presented in Table 3. Both parameters do not indicate some special difference for the specimens failed along bedding planes or those failed across intact rock, but the values of E corresponding to the first and to the third group are a little larger than the values of the second group. All these values are lower than those obtained from conventional laboratory tests (Ferrer, 1990).

Table 3. Mean values for Young's modulus and Poisson relation obtained from the laboratory tests.

| | E (MPa) | $\nu$ |
|---|---|---|
| 1st group | 840 | 0.24 |
| 2nd group | 320 | 0.31 |
| 3rd group | 711 | 0.20 |

In spite conclussions with respect to the behaviour of the material can not be obtained on considering the values of E and $\nu$ separately, if they are put toghether into a graph (Figure 7), the same three different groups can be observed again denoting the different brittle-ductil behaviours.

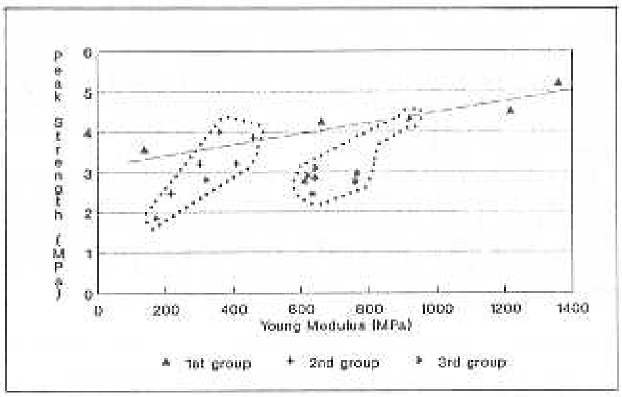


Figure 7. Relations between peak strength and Young's modulus for the tested specimens.

As general conclussions from the tests, next points can be pointed out:
- for the interpretation of the results, the most suitable criterion has been the shape of the stress-axial strain curves and the measured values of axial strain at their different stages
- the results represent a transition between rather brittle failure and pseudo-plastic or ductile-like behaviour, which can also be observed in the generated failure surfaces of the specimens
- the specimens failed through intact rock (when the load-bedding angle is near 90°) present a more ductile behaviour and lower values for peak strength than those failed along a bedding plane
- the complete failure of the specimens of 2nd and 3rd groups at the peak strength imply the material behaves as a "granular" material (c = 0) in the post-peak portion, without failing across a determined failure plane
- before the yield strength, the behaviour of the rock follows an elastic model with an average Young's modulus of 625 MPa

Two main aspects must be considered, then, in the analysis of the behaviour of weak rock masses:
- strains play a main rol in their deformational processes
- plastic behaviour and low strength (even lower than for joints) are to be expected when failure take place through intact rock.

## 4 ANALYSIS OF THE BEHAVIOUR OF THE MATERIAL CONSIDERING THE STRESS-STRAIN RELATIONS

Once some conclussions were stablished with respect to the stress-strain relations of the material, the rock mass was modelized to carry out the analysis of the slope by the finite element method (Zace Services, Ltd., 1989). This method allows the consideration of these relations as well as the use of plastic failure criteria, aspects that are not considered by other conventional methods.

The objective was the performance of the back-analysis to reproduce the failure process. For this, the initial parameters were those obtained from the tests, and they were little by little varied until the modelization of the main steps of the behaviour of the slope was obtained. Water pressure and in situ tectonic residual stresses were also introduced in the analysis.

The behaviour law use to modelize the material was the Drucker-Prager criterion (1952), which is one of the more extended plastic criteria to use in rock mechanics.

Before the back-analysis, some other analysis were carried out considering:
- geomechanical parameters obtained from conventional laboratory tests, and
- geomechanical parameters obtained from the back-analysis by limit equilibrium methods,

but it was no possible to modelize the deformational process of the slope and to reproduce the field observations along the failure process.

So the performance of the back-analysis was decided trying to reproduce the failure model and mechanism and looking for the deformational parameters acting at the beginning of the failure (safety factor = 1.0).

In the initial model all the characteristics of the rock mass were introduced, and for the material E and values obtained from the tests were used. After play with the values of c and o for the fault and the intact rock, the correct modelization of the failure process was obtained, with the same deformational steps observed in the slope, mainly those affecting to the upper part and to the toe of the slope.

The obtained results are included in Table 4. In the iterative process, the non convergence of the calculations indicate that the plastic strains have been reached and, according to the chosen failure criteria, the failure of the material begins. The nodal displacements in Table 4 correspond to the maximun value obtained for each one of the safety factor steps.

Table 4. Results obtained from the back-analysis of the slope.

| Step (S.F.) | 0.94<br>c | 0.96<br>c | 0.98<br>c | 1.00<br>c | 1.02<br>nc | 1.04<br>nc |
|---|---|---|---|---|---|---|
| Max.Displ. ($m \times 10^{-3}$) | 9.30 | 9.70 | 10.3 | 11.4 | 15.2 | 150 |

c = the yield point has not been reached
nc = yield surface has been reached

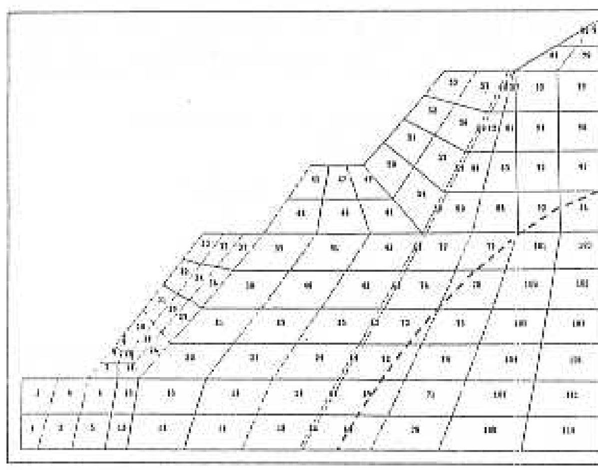

Figure 8. Modelization and initial mesh of the slope.

The initial mesh is reproduced in Figure 8, and Figure 9 shows the deformed meshes for steps with safety factor = 1.0 (inmediately previous step to failure), S.F.= 1.02 (were the failure begins) and S.F.= 1.04, were the plastic displacements have been surpassed.

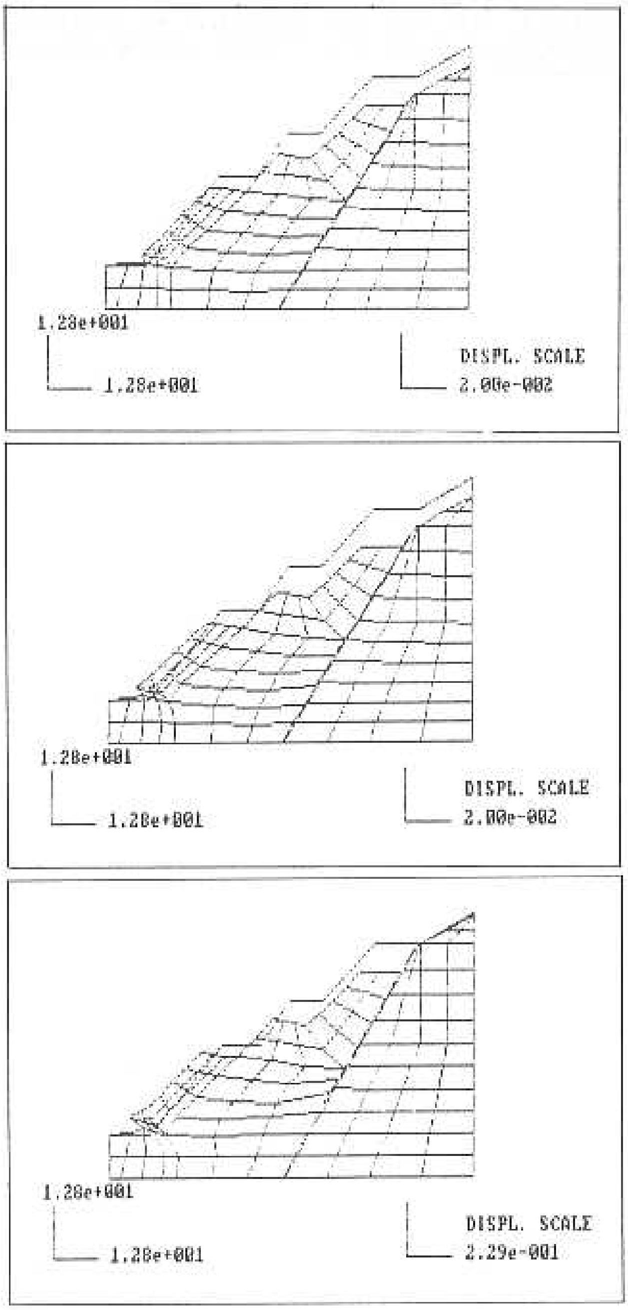


Figure 9. Deformed meshes for the step previous to the failure, for the step when the failure begins (plastic displacements are reached in the failure surface zone and in the toe of the slope) and for the step after the failure.

The modelized deformations agree with the field observations, being clearly represented the tension crack, coinciding with the fault, and the flexion and failure of the toe of the slope.

In the first steps of the analysis (when the stress-strain relations are elastic and the slope is stable), maximun displacementes occur at the upper part of the slope, while compresive stresses increase in the inner and at the toe. When the no

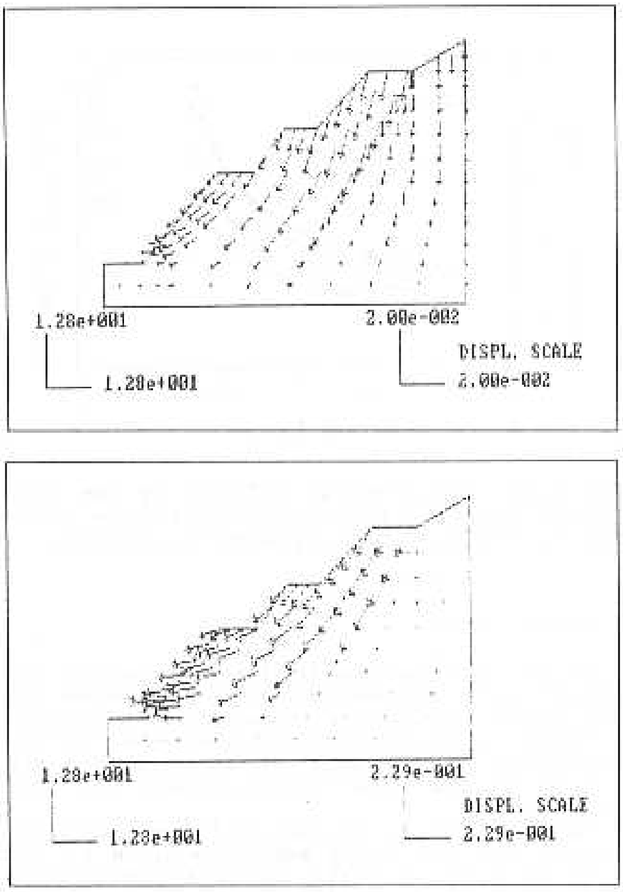


Figure 10. Displacements vectors for the analysis steps corresponding to elastic and plastic displacements (when the failure begins).

convergence is reached (that is, generalized plastic deformations take place), maximun displacements appear also at the toe's slope zone, implying plastic displacements through the intac rock; in this step is when the failure take place and the general instability of the slope occur.

Figure 10 shows the displacement vectors at the beginning of the analysis (for elastic displacements) and for the step when the failure is reached (F.S.= 1.02). These vectors indicate the type of movement affecting to the rock mass and the location of the failure surface; the portion of the rock mass which was not affected by the slide can also be seen, corresponding with what really happened.

In order to know more about the behaviour of the intact rock, some representative nodes located along the failure surface zone were chosen to study their displacements (see Figure 11). All of them were located along the inferior part of this surface, and near the toe of the slope. Figure 11 shows that the plastic displacements are reached in a gradual way, though in the last elastic steps the increments are a little larger. After the non elastic displacements occur (F.S.= 1.02), the values are much more larger: there in no relation between the applied load and the produced displacements.

With respect to the internal stresses generated in the rock mass along the deformational process, tensional stresses appear along the fault at the step previous to the failure, increasing when the failure begins. Along all the process, compresive stresses are increasing in the interior of the slope, which decrease when the failure occur; at the toe zone the stresses decrease as the deformational process go on.

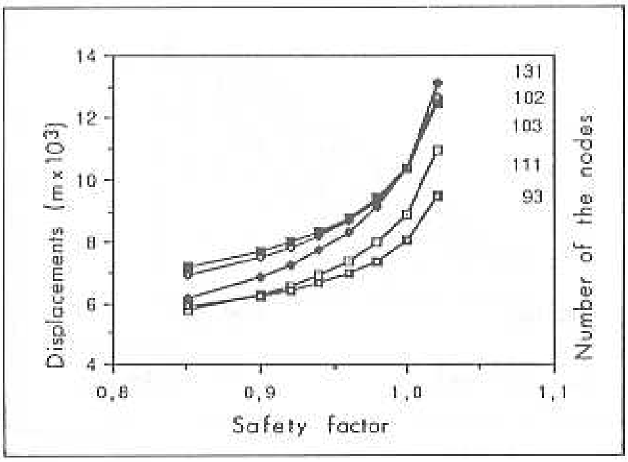


Figure 11. Displacements suffered by the nodes located along the failure surface through intact rock for the different steps of the analysis.

5. CONCLUSSIONS

- The deformational and failure processes of the studied weak rock mass can not be explained and interpreted with the use of conventional strength parameters and analysis methods; the failure mechanisms of this type of materials doesn't respond to the more usual models for rock mechanics.
- The ductile-like or pseudo-plastic behaviour deduced from the study and observation of the behaviour of the rock mass, can not be modelized by failure criteria (commonly used for brittle materials) which suppose the failure beggins at the peak strength.
- From the data obtained with non conventional laboratory tests (with a stiff servo-controlled testing machine), different behaviours for the material can be deduced when it fails along a joint or through intact rock; these different behaviours are deduced from the values of the axial strains suffered by the specimens and by the shape of the stress-axial strain curves, which denote the influence of the strain rate and of the stress-strain relations on the behaviour and strength of the rock.
- The stress-axial strain curves obtained for specimens that didn't fail along existing joints, denote a plastic behaviour associated to the failure through intact rock.
- From the laboratory tests data, an alternative failure criteria can be deduced, considering that the failure beggins when plastic deformations are reached, that is, at the yield point.
- When these previous aspects are considered in the analysis of the rock mass, and adequate stress-strain methods and failure criteria are used, its deformational and failure process can be modelized, and representative failure mechanisms and behaviour rules of the material answering to the real behaviour are obtained.

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